

A-level Physics Tutor Guides

A-level Physics
COURSE NOTES

QUANTUM PHYSICS



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Quantum Theory

Quantum Theory summary

Quantum theory is used to explain interactions between matter and energy on the extremely small scale of atoms and sub-atomic particles.

basic points:

- energy comes in small, discrete units called photons
- elementary particles behave both like particles *and* like waves
- particle motion is random.
- uncertainty: particle position and momentum cannot be known at the same time

Planck's Quanta

Max **Planck**(1901) first proposed the idea that light was emitted as discrete packets of energy called **quanta**. He also showed that each packet (**quantum**) had energy given by the equation:

$$E = h\nu$$

where,

E = energy (J)

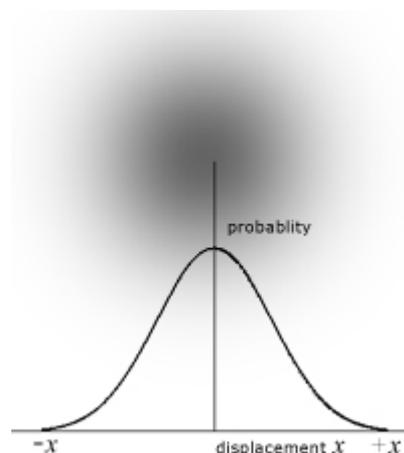
h = Planck's constant, 6.626×10^{-34} Js

ν (nu) = frequency of light radiation (Hz, s^{-1})

However, **Einstein**, as part of his photo-electric effect theory, described these quanta as streams of particles, which he termed **photons**.

Wave-particle duality

It is a common mistake to think of particles as 'matter waves' or 'energy waves'(like photons).



The wave properties of particles relates to their statistical position about a point. Rather than being like a billiard ball, a particle can be considered to be a fuzzy ball, where the depth of colour corresponds to position. So the particle spends most time at the centre of the fuzziness and least time at the edges.

Regarding the reality of particle duality, the **Copenhagen interpretation** is meant as clarification on the issue.

Essentially, the statement reads, **nothing is real unless it is observed**. So a point of matter can be considered as a wave or a particle. If it behaves like a particle then it *is* a particle. If it behaves like a wave, then it *is* a wave.

...more on this in the section 'wave-particle duality'

Heisenburg's Uncertainty Principle

For a particle, position and momentum cannot be known at the same time.

Mathematically the product of the uncertainty in the position of a particle (Δx) and the uncertainty in the momentum (Δp) could never be less than a particular value. That particular value is related to Planck's constant h .

$$\Delta x \Delta p \leq \frac{h}{2}$$

$$\text{where } \hbar = \frac{h}{2\pi}$$

The Wave Function

The wave nature of matter is described in some detail by the **Schrödinger Wave Equation**. This is used to predict the future behavior of a dynamic system. Note, the waves referred to in the equation are 'probability waves'.

This is the Schrödinger wave equation for a free particle in one direction. The symbol Ψ is called the wave function.

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} = i\hbar \frac{\partial \Psi}{\partial t}$$

The wave function is used to give information on:

- probability distributions in 1D, 2D or 3D
- quantum states
- energy levels

There are other versions of the wave equation to cover different situations.

One version of the equation predicts where electrons exist around an atomic nucleus. Electrons are found in discrete 3D regions called 'orbitals'. Much of this work was done by **Wolfgang Ernst Pauli** in his famous 'exclusion principle'.

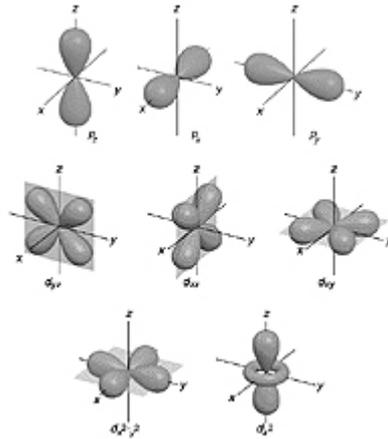


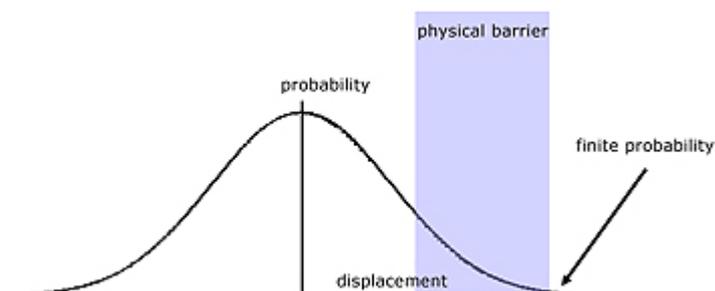
image courtesy of Chemistry Dept. University of Florida

Other wave solutions refer to individual particles, groups (beams) of particles, atoms and molecules.

Quantum Tunnelling

This is a phenomenon relating to the probability wave of an electron and its range over an insurmountable barrier.

In classical terms, the electron does not have enough kinetic energy to jump a potential energy barrier. Compare a bouncing ball not able to bounce over a brick wall, which is too high.



Electrons are particles of matter that exhibit a wavelike quality. According to quantum mechanical theory the wave function (probability wave) of the electron extends across the barrier.

So there is a finite possibility of it existing there.

Quantum tunnelling is used in tunnelling diodes and quantum tunnelling composites(QTC's).

Tunnelling diodes are important elements in high frequency devices like CRO's, frequency generators and detectors.

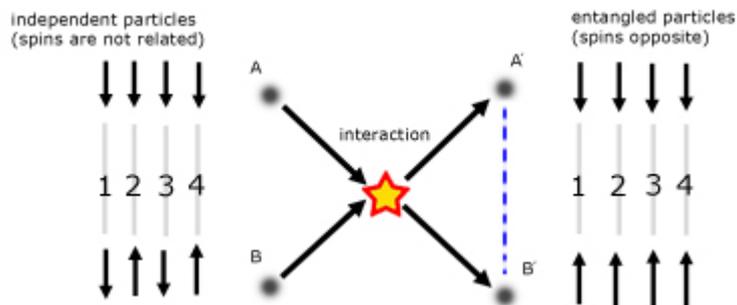
Quantum tunnelling composites is the name given to the technology of 'touch screens'. These have applications in phones, TV's, cameras and monitors.

Quantum Entanglement

Quantumly entangled particles retain a connection between them even if separated across vast distances. So if one particle is disturbed, the other is also disturbed.

Einstein is said to have described the phenomenon as "spooky action at a distance" (a measure of his disbelief on the subject).

As an example, consider two electrons that through an interaction become entangled.



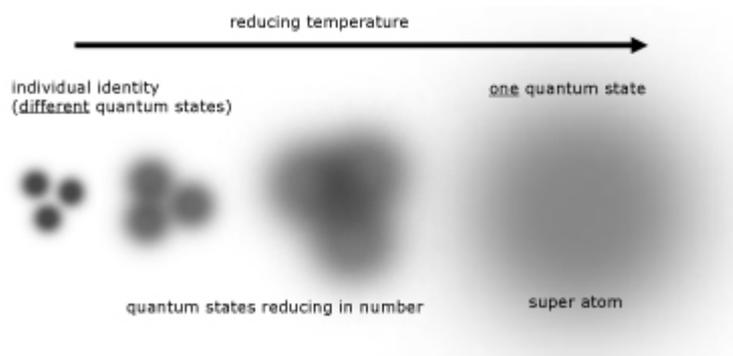
Each number in the diagram represents a run of the experiment, when the spins of particles A and B are measured.

The result of the entanglement is that spin states of the two particles become complementary. One state will be 'up' while the other is 'down'.

The shared state means that a change to one electron is reflected in the other. This has immense potential for the development of 'quantum computers' - computers that will be much faster than those at present.

Bose-Einstein Condensate (BEC)

At temperatures approach absolute zero, the atoms in a gas are in their lowest energy level. At these temperatures quantum mechanical effects become more apparent. As a result of their probability positioning, atoms can be thought of as fuzzy balls. When the temperature decreases, the balls spread out and become less defined. Eventually a point is reached when their fuzziness(where they are positioned) overlaps. At this time atoms lose their individual identities. They all have the same quantum state and coalesce into a single 'super atom'.



The properties of condensates are quite bizarre. These include:

- increased optical density(enough to slow photons down to walking speed!)
- super fluidity(absence of viscosity)

Wave-Particle Duality

The De Broglie Equation

In 1924 **Louis de Broglie** proposed that matter had a wave nature. His famous equation links the momentum 'p' of a particle with its wavelength 'λ' (the **De Broglie wavelength**).

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Note, these are not matter waves. The amplitude of the wave at a point represents the probability of the particle's position.

The relation was verified by **Davisson** and **Germer** in 1927 by electron diffraction in crystals.

...more on this in the section 'electron diffraction'

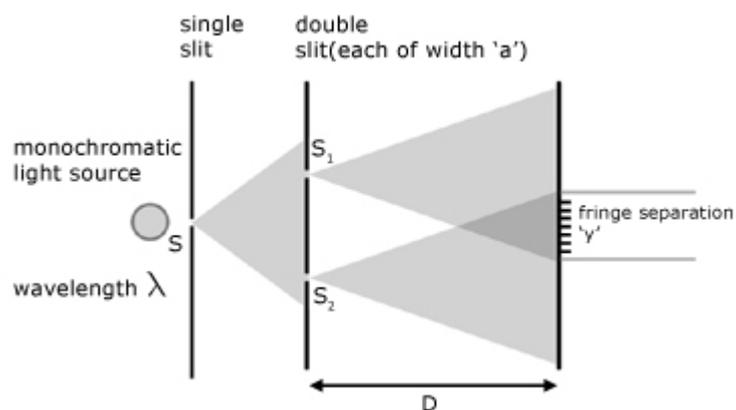
Optical interference from a double slit

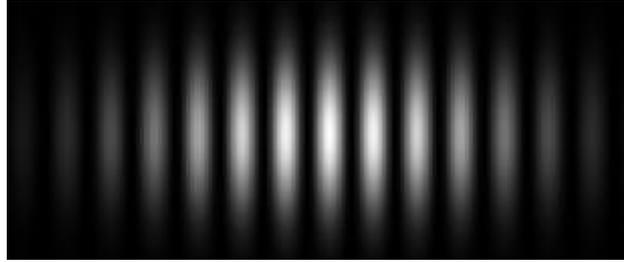
This was first demonstrated by **Thomas Young** in 1801.

Two light sources can produce interference provided:

1. the sources are coherent*
2. the wave amplitude is approx. the same

* a constant phase difference must exist between each wave (so the frequency must be the same).





Young's double-slit equation is:

$$\lambda = \frac{ay}{D}$$

λ wavelength

y fringe separation adjacent bright/dark fringes

a slit separation

D perpendicular distance between slit plane and screen

experimental observations

1. fringe separation increases by increasing separation 'D'
2. fringe separation decreases by decreasing 'a'
3. increasing the slit width 'a' increases fringe brightness but blurring occurs
4. moving the single slit (S_1) closer to the double slit ($S_1 S_2$) increases fringe brightness with no effect on separation

Quantum effects

Quantum mechanics explains the fringe pattern from particle path probabilities.

The position of particles is described mathematically by probability waves. As in the classical case, amplitudes add and subtract from one another. An interference pattern is the result.

Considering photons as particles, if we reduce the emission rate, photons go through the slits one at a time*, and after enough time has elapsed, an interference pattern is built up.



*Strictly speaking, according to quantum mechanics, a photon can go through BOTH slits at the same time.

Similar interference patterns are obtained by firing particles(eg electrons) at a double slit. The bottom line is that waves behave as particles and vice versa.

Wavelengths compared

X-rays waves have a wavelength of the order of 10^{-11} m travelling at the speed of light at approximately 3×10^8 ms⁻¹.

We can get an idea of wavelength for particles, but it must be remembered that their wavelength depends on velocity, which is variable.

So as an example let's take an electron travelling at 10% the speed of light.

$$m_e = 9.1 \times 10^{-31} \quad v = 3 \times 10^7 \text{ ms}^{-1} \quad h = 6.6 \times 10^{-34} \text{ J.s}$$

using the De Broglie equation,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\lambda = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^7}$$

$$= \underline{2.4 \times 10^{-11} \text{ m}}$$

The electron wavelength travelling at 10% the speed of light has approximately the wavelength of X-rays. Since wavelength and velocity are inversely proportional, the higher the velocity, the shorter the wavelength. Thus shorter wavelengths, approaching that of gamma rays, can be obtained with higher electron velocities.

Using more massive particles, for example neutrons, produces even shorter wavelengths. Neutrons are approximately 2000 times the mass of electrons. So their wavelength for the same velocity, will be 1/2000 th (approx 1.4×10^{-14} m).

There is another wavelength pertaining to particles that is different from the De Broglie wavelength. This is called the **Compton wavelength**.

The Compton wavelength has particular relevance in quantum mechanics. It was introduced by Arthur Compton as part of his theory on the scattering of photons by electrons (**Compton scattering**).

By definition, the Compton wavelength of a particle is the wavelength of a photon with the same energy as the rest-mass energy of the particle.

$$h\nu = m_0c^2$$

but $c = \nu\lambda$ and $\nu = \frac{c}{\lambda}$

substituting for ν

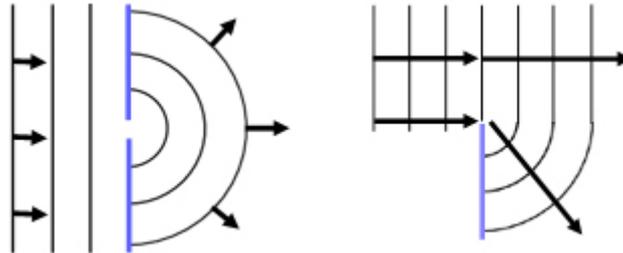
$$h\frac{c}{\lambda} = m_0c^2$$

$$h = m_0c\lambda$$

$$\lambda = \frac{h}{m_0c}$$

Electron Diffraction

What is diffraction?

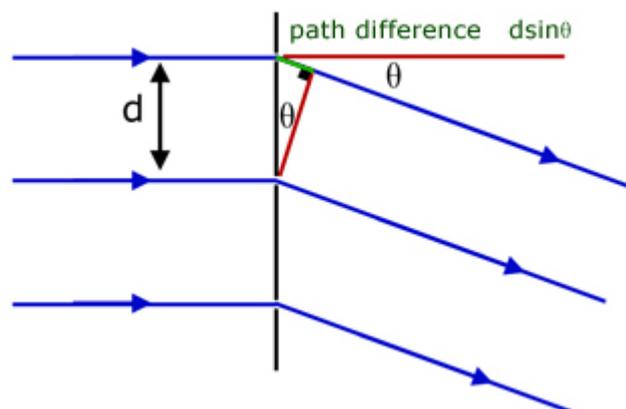


Classical physics gives us a definition of diffraction in terms of waves. It is the phenomenon whereby light or sound waves bend around small obstacles or spread out through small openings.

The Diffraction Grating

A diffraction grating is an array of identical, equidistant, parallel lines on a surface. Gratings are used to produce optical spectra from a single source, parallel beam of light. There are two types of grating: reflection & transmission

The transmission grating:

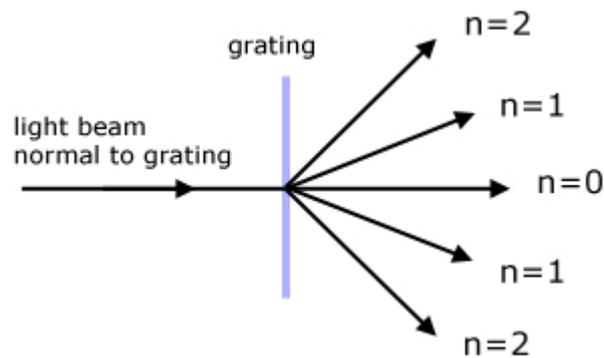


Light rays are diffracted at the edge of each gap in the grating. The waves of light add when the path difference is a whole number of wavelengths. So discrete images at different angles are produced.

$$d \sin \theta = n\lambda \quad (n = 0, 1, 2, \dots)$$

'n' values correspond to:

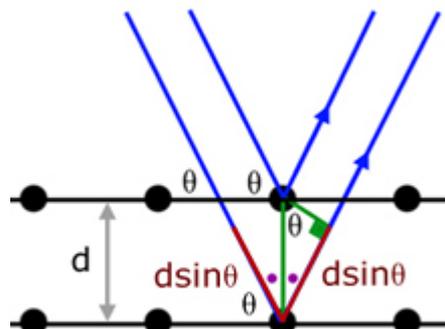
- n=0 no path difference
- n=1 one wavelength path difference
- n=2 two wavelengths path difference etc.



Bragg Diffraction (an example of reflection diffraction)

Bragg diffraction occurs when particles or waves with wavelength ' λ ' comparable with atomic spacing ' d ', interact with atoms in crystals. At a particular angle θ^* waves are diffracted by atoms in adjacent planes and interfere constructively. As with transmission diffraction gratings, a bright image is only produced when the path difference is a whole number of wavelengths.

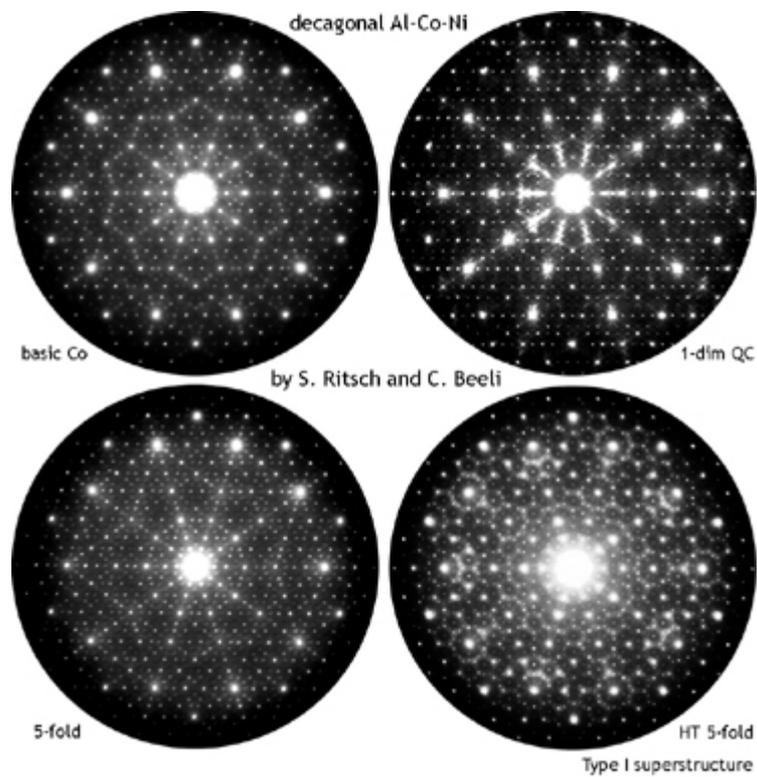
* the angle the incident wave makes with a plane of atoms



$$n\lambda = 2d\sin\theta$$

Practically, the incident waves can be provided by X-rays, electrons or neutrons.

Electron diffraction interference patterns



Electron, X-ray and neutron diffraction compared

The most commonly used method is X-ray diffraction.

Summary:

	electron	x-ray	neutron
scattering	by electrostatic repulsion of nucleus	by electron cloud around nucleus	by interaction with the nucleus
resolution	moderate	moderate	high
penetrating power	poor (requires thin specimens)	good	good
matter interaction	high (unreliable results)	none	moderate
magnetic effects	no	no	yes (neutrons have their own magnetic field)
good for light elements	no	no	yes
particular uses	crystals	crystals	fuel rods, archaeological artefacts

The Photo-Electric Effect

History

Hertz discovered the photo-electric effect in 1887. His experiment concerned two parallel metal electrodes, with a p.d. across them. He observed that when ultra-violet light was shone on the electrodes an electric spark jumped between them. Many of the properties of the effect were established by **Hallwachs** and **Lenard** in the years leading up to Einstein's theory in 1905.

The Photon Model

Max **Planck**(1901) first proposed the idea that light was emitted as discrete packets of energy called **quanta**. He also showed that each packet (**quantum**) had energy given by the equation:

$$E = h\nu$$

where,

E = energy (J)

h = Planck's constant, 6.626×10^{-34} Js

ν (nu) = frequency of light radiation (Hz, s^{-1})

However, **Einstein**, as part of his photo-electric effect theory, described these quanta as streams of particles, which he termed **photons**.

Observations

1. Emission of photo-electrons only occurs if the frequency of incident radiation exceeds a minimum value called the '**threshold frequency**'.
2. The emission of photo-electrons starts immediately the surface becomes irradiated(provided the frequency of the radiation is above the thresh-hold frequency).
3. If the incident radiation has a frequency above the thresh-hold frequency:

$$(\text{no. electrons emitted/sec.}) \propto (\text{radiation intensity})$$

4. Increasing the frequency of the incident radiation has the effect of increasing the kinetic energy of **all** emitted electrons. Hence the maximum kinetic energy an electron may have is increased by increasing the frequency.
5. Radiation intensity is **independent** of electron kinetic energy. The kinetic energy of electrons is solely controlled by the radiation frequency.

Wave Theory predictions

Classical wave theory predicts that energy is carried in the wave front. Electrons absorb the energy from the wave until the level exceeds the work function*. Electrons are then released from the surface of the metal.

*W (or Φ) minimum energy to remove an electron

Three main predictions come from this classical explanation:

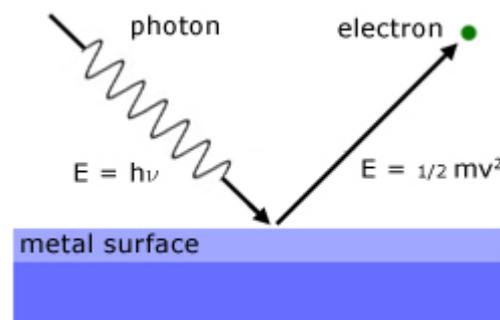
1. Radiation intensity is proportional to the resulting maximum electron kinetic energy.

$$(\text{radiation intensity}) \propto (\text{maximum electron kinetic energy})$$

2. The effect should occur for any frequency of light

3. There is a delay between the radiation contact with the surface and the first release of electrons.

Einstein's Photo-Electric Effect Theory



The theory is based on a beam of light being considered as a stream of photons, each with energy $h\nu$.

Before explaining the theory in more detail, it is important to appreciate what is meant by '**the intensity of light**'. Consider photons emitted from a point source. The sphere around the point, where the photons arrive, enlarges with distance from the point. So at large distances the photons are spread out over a large area.

Note there is **no** diminution of photon energy, whatever the distance travelled.

$$\text{Intensity (I) beam of light} \propto (\text{no. photons}/\text{m}^2/\text{sec.})$$

Einstein's *big idea*, in essence, was that when a photon collides with an electron there are two possible outcomes:

1. the photon reflects from the electron with no energy transfer
2. the photon is absorbed by the electron and gives up ALL its energy to it

Photons are paired with electrons. So there is no question of one photon sharing energy with more than one electron.

intensity of light \propto (no. electrons emitted by a surface)

or more accurately,

(no.photons/m²/sec.) \propto (no. electrons/m²/sec.)

So when a photon arrives at the surface, an electron is emitted. This explains the instantaneous nature of the phenomenon.

The photon energy received is used to overcome the forces holding the electron within the surface and to give it kinetic energy to escape.

Einstein's photo-electric effect equation describes this process in more detail:

$$h\nu = W + \left(\frac{1}{2}mv^2\right)_{\text{MAX}}$$

Note this involves the maximum electron kinetic energy. Many electrons emerge from the surface with less than the maximum energy. This is a result of energy lost in collisions before they are free.

W is called the '**work function**' of the surface. It is a measure of the energy that binds each electron into the surface. The constant is different for different materials.

That there must be a minimum frequency for electron emission is implicit.
If

$$h\nu < W$$

then there is not enough energy available to release an electron.

However, that particular frequency for an electron to be released is given by:

$$h\nu_0 = W$$

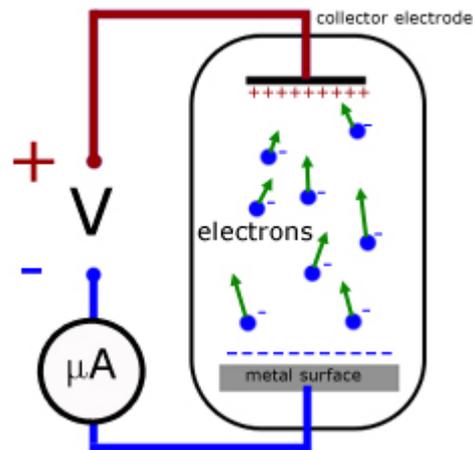
where ν_0 is the threshold frequency.

Using $c = \nu_0\lambda_0$ and substituting for $\nu_0 = c/\lambda_0$, the threshold wavelength λ_0 is given by:

$$h \frac{c}{\lambda_0} = W$$

$$\lambda_0 = h \frac{c}{W}$$

Millikan's apparatus - **stopping potential**



In 1916 Millikan devised a series of experiments that completely vindicated Einstein's theory. Using monochromatic light, the apparatus tested three surfaces in turn (lithium, sodium and potassium). He found that altering the potential between the surface and the electron collector electrode, altered the 'electron current'.

When a small positive potential was applied to the metal, only electrons with enough KE escaped to impact on the collector electrode. The remainder were pulled back to the surface. So only a small electric current was detected at the electrode.

With increased +ve potential, the electron current was reduced.

Eventually there came a point when the current reduced to zero. This corresponded to electrons with the maximum KE being stopped from reaching the collector. This p.d. between both electrodes is called the **stopping potential V**.

We can now integrate 'stopping potential' into the photo-electric effect equation.

The work done by an electron in moving against the 'stopping potential'(ie against the 'electric field') is equal to the maximum KE of the electron.

$$eV = \left(\frac{1}{2} mv^2 \right)_{\text{MAX}}$$

from Einstein's photo-electric effect equation:

$$h\nu = W + \left(\frac{1}{2} mv^2 \right)_{\text{MAX}}$$

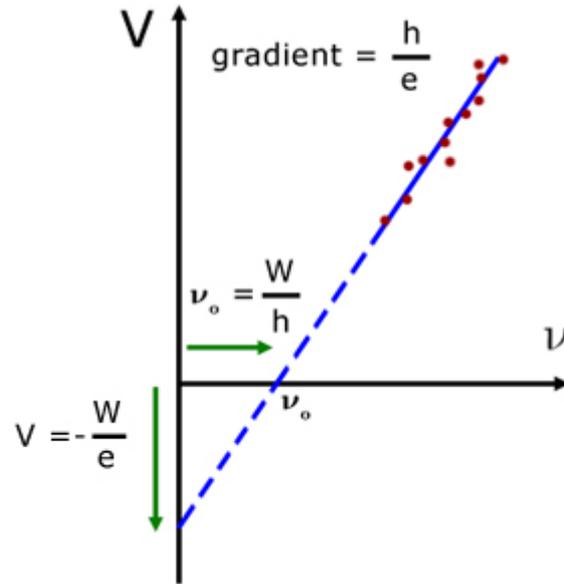
substituting for KE,

$$h\nu = W + eV$$

rearranging into the form for a straight line 'y = mx + c'

$$V = \frac{h}{e}\nu - \frac{W}{e}$$

For different materials, a plot of V against frequency ν is a straight line, with intercepts on the V-axis and the ν -axis.



intercept on V-axis ($\nu = 0$) is:

$$V = -\frac{W}{e}$$

intercept on the ν -axis ($V = 0$) is:

$$\nu = \nu_0 = \frac{W}{h}$$

where ν_0 is the threshold frequency

gradient is given by :

$$m = \frac{h}{e}$$

notes

1. for different materials the graph has the same gradient but different intercepts
2. measurements of W (the work function) can be made
3. measurements of h (Planck's constant) can be made

Energy Levels & Ionisation

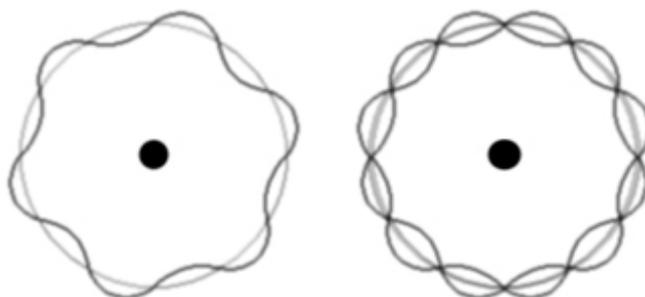
Visualising energy levels

In an atom, electrons around a central nucleus can only have particular energy values. These discrete values are termed '**energy levels**'.

In a diagram they are represented by horizontal lines, with the lowest level (the **ground state**) at the bottom and the highest level (**ionisation**) at the top.

The levels are a consequence of the wave nature of electrons, as described by quantum mechanics. Around an atom an electron exists as a particular **standing wave**, with the number of nodes and antinodes dictated by the quantum number 'n' .

As an example take the simplest atom, the hydrogen atom.



The levels are dictated by concentric wave states of an exact number of electron **half wavelengths**.

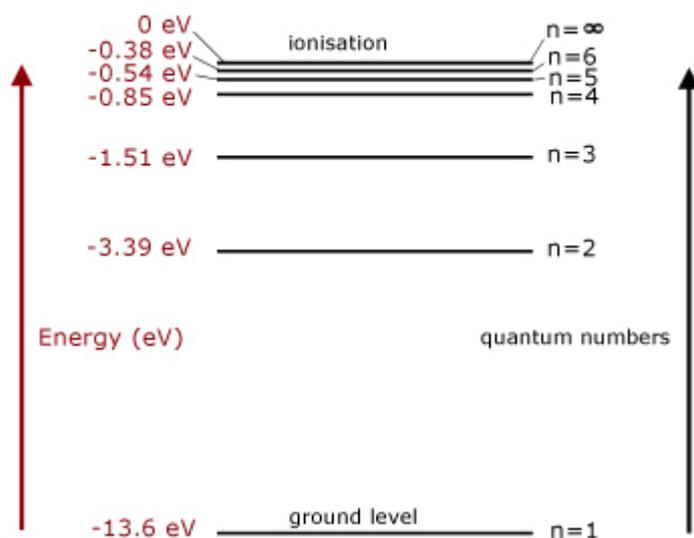
Remember a 'standing wave' (or stationary wave) is the interference between two waves of the same wavelength and speed, travelling in opposite directions.

In the example there are 6 maxima and 6 minima. So the number of half wavelengths is 12.

Therefore the electron state illustrated has $n=12$.

n	no. complete waves
1	$\frac{1}{2}$
2	1
3	$1 \frac{1}{2}$
4	2
5	$2 \frac{1}{2}$
6	3

In the energy level diagram(below), energies have to be measured relative to one another. Remember the ground state is the lowest level and ionisation is the highest. So you would expect the ground state be zero. However, that is not the case.



Remember we are measuring potential energy. This is set to zero at infinite distance from the atomic nucleus or molecule. The state is that of an ionised or free electron.

Relative to this state, all the bound electron states have negative potential energy and are measured below it (because they have less energy).

Note that electron energies are expressed in **electron volts**.

An electron volt (eV) is the kinetic energy acquired by an electron, when accelerated through a p.d of 1 volt. Using $E=QV$ the energy is given by $e \times 1 = 1.6 \times 10^{-19}$ Joules.

Quantum numbers are given to each energy level. The ground state is $n=1$, with the n numbers increasing by '1' for each level. The highest energy level, where energy is zero, has $n = \infty$.

Energy level changes

When radiant energy is absorbed by a bound electron it is **excited** into a higher energy level. Conversely, when an electron falls to a lower level, radiant energy is given out.

Example: an electron moves down from level 3 to level 1 and gives out a photon of light of frequency ν (nu) .

$$E_3 - E_1 = h\nu$$

substituting for ν from $c = \nu\lambda$ and making λ the subject,

$$E_3 - E_1 = h \frac{c}{\lambda}$$

$$\lambda = \frac{hc}{E_3 - E_1}$$

Spectral groups

Looking more closely at the spectrum of hydrogen it is found to be composed of distinct spectral groups.

To understand this we must look in detail at energy level changes. For simplicity we will look at levels from $n=4$ to levels below.

If an electron is excited up from the ground state ($n=1$) to level 4, there are four possible paths for it to come back down again to the original state.

4	4	4	4
3	3		
2		2	
1	1	1	1

If we analyse these transitions/jumps, it is found that they can be divided further into six basic types:

4	4	4			
3			3	3	
	2		2		2
		1		1	1

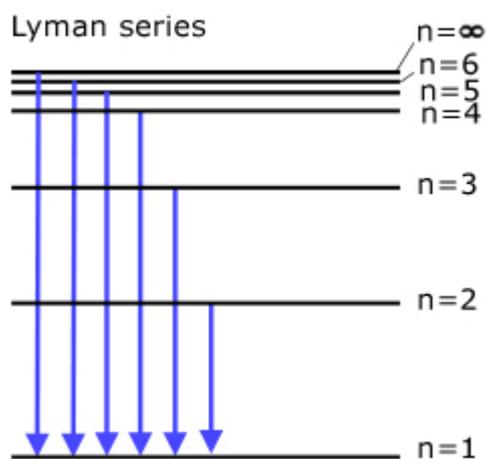
The spectrum of hydrogen contains many distinct sets of emission lines. Not all of these are transitions to the ground state or are between intermediary levels.

Three groups of transitions fall between many levels to one particular level in one jump.

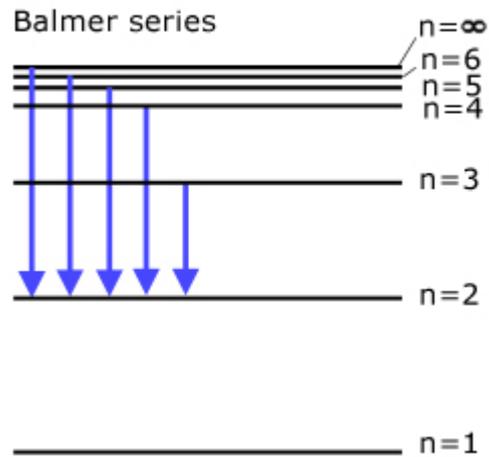
These are called:

The Lyman series (all transitions directly to ground state)

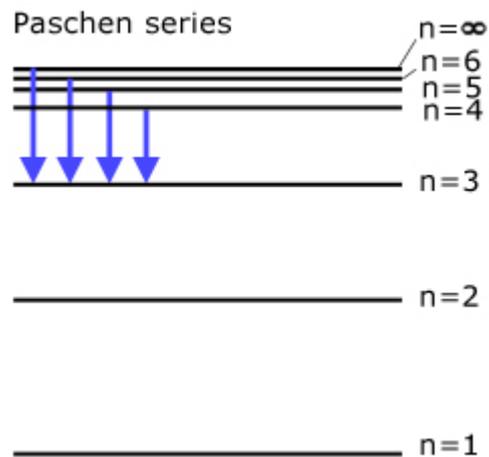
Light is emitted in the ultra-violet.



The Balmer series (all transitions directly to state $n=2$)
Light is emitted in the visible spectrum.



The Paschen series (all transitions directly to state $n=3$)
Light is emitted in the infra-red.



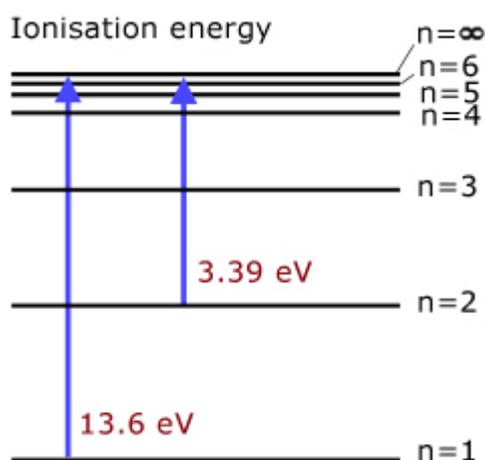
Ionisation

In the ground state, where $n=1$, the electron is not excited and has the least energy.

Remember in energy level diagrams energy is measured upwards. So for electrons reaching higher energy levels it will take less energy to remove them as they are closer to the highest level (ionisation).

The minimum amount of energy to release an electron from the ground state is called the **first or principal ionisation energy**. The release of electrons in higher states are named as their quantum numbers. For example the second ionisation energy (from level $n=2$), third ionisation state (from $n=3$) etc. These principal ionisation energies become smaller and smaller with increasing quantum number, as less energy is required to make up the difference in energy to produce ionisation.

examples:



the 1st ionisation energy of hydrogen is 13.6 eV
(level $n=1$ to ionisation level $n=\infty$)

the 2nd ionisation energy of hydrogen is 3.39 eV
(level $n=2$ to ionisation level $n=\infty$)

Excitation Potentials and Energies

Excitation potential is defined as the minimum potential required to excite a free neutral atom from its ground state to a higher state.

Think of a free electron accelerated through a potential and giving up all its energy to an electron in an atom, taking it to a higher energy state. The accelerating p.d. is the excitation potential for that energy change.

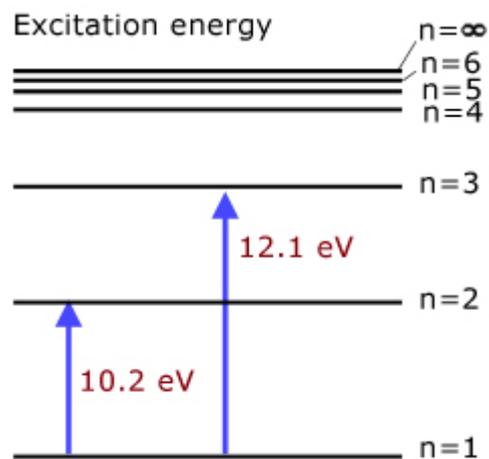
examples:

1st excitation potential of hydrogen 10.2 V
(level $n=1$ to level $n=2$)

2nd excitation potential of hydrogen 12.1 V
(level $n=1$ to level $n=3$)

Excitation energy is the energy required to excite an atom from its ground state to a higher state.

examples:



1st excitation energy of hydrogen 10.2 eV
(level $n=1$ to level $n=2$)

2nd excitation energy of hydrogen 12.1 eV
(level $n=1$ to level $n=3$)